Modeling the meteoroid environment far from the ecliptic plane

– and –

Can a tilted plane of symmetry explain seasonal variations in the meteoroid environment?

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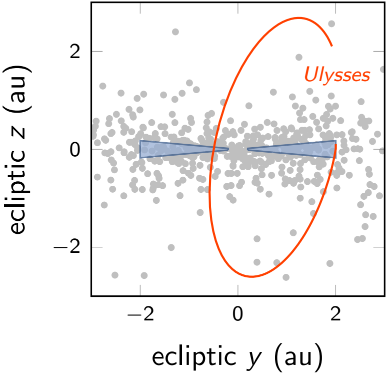
Our meteoroid model (MEM) assumes that the spacecraft/observer lies near the ecliptic plane.

This assumption simplifies the math and decreases run time.
It's fine for most, but not all, spacecraft.

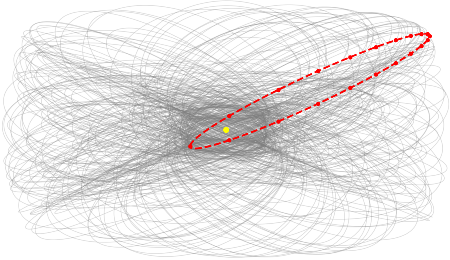


Removing this assumption requires us to calculate two things:

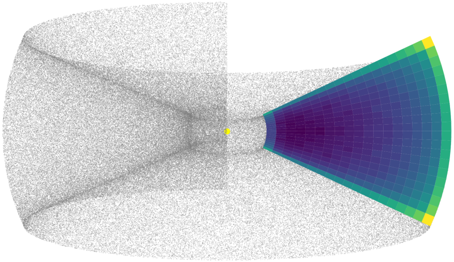
- η: the meteoroid number density as a function of r and z
- $\vec{v}$: the encounter geometry/velocity vector

Meteoroid models often assume that M, ω, and Ω are randomized

The inclination limits the maximum vertical displacement from the ecliptic:



Once averaged over angle, the spatial PDF depends only on r and z:



$$\eta = \frac{1}{2\pi r}\,\mathrm{PDF}(r,z) = \frac{1}{2\pi^3} \times \frac{1}{ra}\frac{1}{\sqrt{(ae)^2-(r-a)^2}} \times \frac{1}{\sqrt{\sin^2 i-(z/r)^2}}$$

We can express the spatial PDF in terms of independent parameters

This PDF[1] has singularities at: $r - a = \pm ae$ and $z = \pm r \sin i$:

$$\mathrm{PDF}(r, z) = \frac{1}{\pi^2} \times \frac{1}{a} \frac{1}{\sqrt{(ae)^2 - (r-a)^2}} \times \frac{1}{\sqrt{\sin^2 i - (z/r)^2}}$$

But, if we express it in terms of $s = (r - a)/ae$ and $\xi = z/r \sin i$, we get:

$$\mathrm{PDF}(s, \xi) = \frac{1}{\pi^2} \times \frac{1 + es}{\sqrt{1 - s^2}} \times \frac{1}{\sqrt{1 - \xi^2}}$$

[1] See also Haug (1958), Kessler (1981)

An analytic CDF allows us to compute realistic meteoroid number densities at **all** locations.

The integral of each PDF component has closed form:

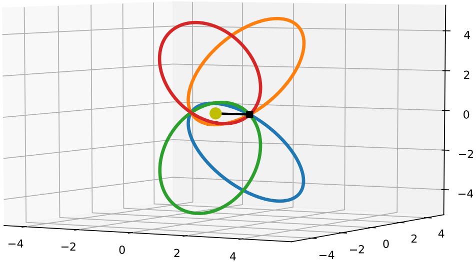
$$\int \frac{1+es}{\sqrt{1-s^2}} = \sin^{-1} s - e\sqrt{1-s^2}, \qquad \int \frac{1}{\sqrt{1-\xi^2}} = \sin^{-1} \xi$$

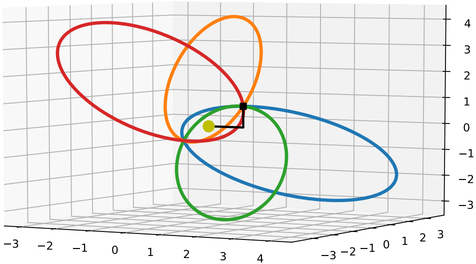
This allows us compute more reasonable number densities:

1. We select a distance d over which we believe the environment to vary.
2. We compute $\Delta s = ae \cdot d$ and $\Delta \xi = r \sin i \cdot d$
3. $\Delta \mathrm{CDF} = [\mathrm{CDF}_s(s+\Delta s) - \mathrm{CDF}_s(s-\Delta s)] \times [\mathrm{CDF}_\xi(\xi+\Delta\xi) - \mathrm{CDF}_\xi(\xi-\Delta\xi)]$
4. $\Delta V = 2\pi r d^2$
5. $\eta = \Delta \mathrm{CDF}/\Delta V$

The orbital elements and location determine the encounter geometry.

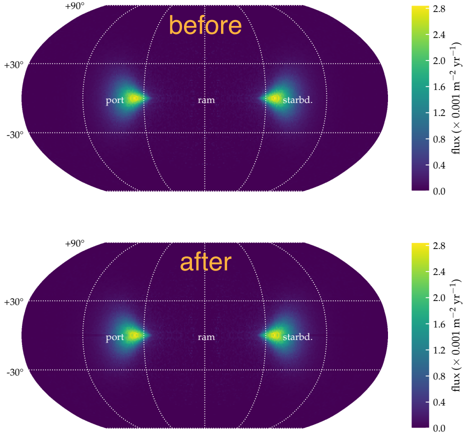
For a given a, e, and i, there are 4 sets of orbit angles that allow the orbit to intersect a given point (provided that i is large enough).

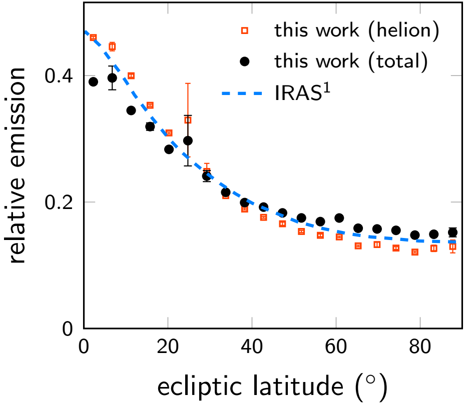




At out-of-ecliptic locations, the apparent directionality is of course no longer symmetric about the ecliptic.

Our updated model matches the zodiacal light emission profile while preserving in-ecliptic results.

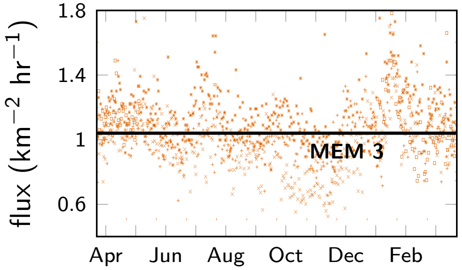




[1]25-μm profile from Nesvorný et al. (2011)

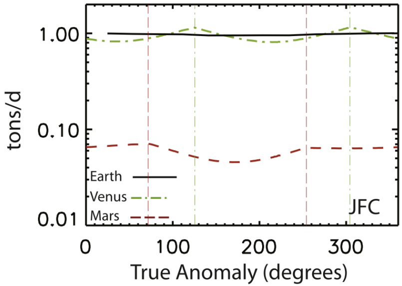
Can a tilted plane of meteoroid symmetry explain seasonal variations?

The meteoroid flux at Earth varies seasonally by about 30-40%; each source has its own pattern.



[1]See Campbell-Brown & Jones (2006)

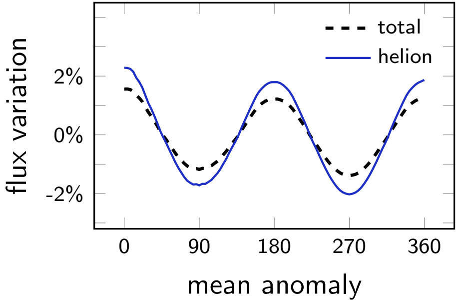
Janches et al. (2020) attributes seasonal variations at Mars/Venus to orbital tilt:



If the meteoroid plane of symmetry is tilted wrt. the ecliptic, could this explain the variations seen at Earth?

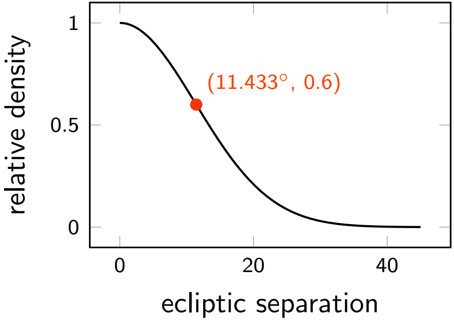
No. Tilting the Earth by 1.5° relative to the meteoroid plane of symmetry produces 1-2% variations.

We tilted the Earth's orbit by 1.5°:



The corresponding change in emission profile is also minimal.

Our helion population has the lowest inclinations, and still requires an 11° tilt for a 40% drop in η.



Summary

We've developed a new version of our meteoroid environment model (MEM 3.1-alpha) that can handle out-of-ecliptic spacecraft or observer locations.

We find that expressing the number density in unitless parameters lets us integrate over singularities.

We've validated the results against past versions and ZC emissions.

This new version allows us to test whether a relative inclination between the meteoroid plane of symmetry and the ecliptic can produce seasonal variations: no.[1] We are unable to reproduce the Janches et al. effect ($\sim 40\%$ variations arising from 1-3° relative tilt).

[1] "Any headline that ends in a question mark can be answered by the word no."
– Betteridge's law